

1. a) $\frac{x}{5} - \frac{y}{10}$ b) $\frac{5}{7d} + \frac{8}{14d}$ c) $\frac{5}{a^2} + \frac{3}{a}$ d) $\frac{a}{b^3} - \frac{c}{b^2}$
 e) $\frac{x}{3k} - \frac{y}{6k}$ f) $\frac{2x}{y^2z^2} + \frac{3}{yz}$ g) $\frac{a}{7x} + \frac{b}{14x}$ h) $\frac{4}{x} - \frac{2}{x^2}$

2. a) $\frac{5x}{2} + \frac{3y}{6} - \frac{2z}{3}$ b) $\frac{5}{4x} + \frac{3}{5x} - \frac{7}{20x}$ c) $\frac{5}{x^2} - \frac{2}{x} + \frac{3}{x^3}$
 d) $\frac{4a^2}{9} + \frac{2b^2}{6} - \frac{c^2}{18}$ e) $\frac{4}{8d} - \frac{2}{6d} + \frac{9}{24d^2}$ f) $\frac{3}{9m} - \frac{2}{18mn} + \frac{3}{6n}$

3. a) $\frac{x+1}{3} - \frac{2x-4}{2} + \frac{3x-2}{6}$ b) $\frac{3a-4b}{5} - \frac{8a-7b}{2} + \frac{3b-4a}{10}$
 c) $\frac{5a}{a^2-b^2} - \frac{3}{a+b} + \frac{5}{a-b}$ d) $\frac{8}{4x^2-9y^2} - \frac{2}{2x-3y} + \frac{5}{2x+3y}$
 e) $\frac{5}{a^2-b^2} + \frac{3}{a-b} - \frac{2}{a+b}$ f) $\frac{4}{p^2-q^2} + \frac{4}{p+q} - \frac{2}{p-q}$

4. a) $\frac{x}{a} - \frac{y}{b}$ b) $\frac{x}{m} + \frac{y}{n}$ c) $\frac{7}{k} + 4$
 d) $5 + \frac{1}{r}$ e) $\frac{x}{y} - 1$ f) $\frac{1}{p} - \frac{1}{q}$

5. a) $\frac{3}{x} + \frac{2}{y} - \frac{1}{z}$ b) $\frac{4a}{3} + \frac{5b}{2} - \frac{7}{8}$ c) $\frac{7}{x} + \frac{2}{y} - 1$
 d) $\frac{2x}{5p} + \frac{3y}{4q} - \frac{5z}{r}$ e) $\frac{5x}{8} - 1 + y$ f) $\frac{2a}{5} + \frac{2b}{3} - \frac{c}{6}$

6. a) $\frac{2a+3b}{5} - \frac{3a-5b}{4}$ b) $\frac{5x-3y}{2a} - \frac{2x+3y}{b}$ c) $\frac{a}{x+1} - \frac{b}{x-1}$
 d) $\frac{5}{2d-e} + \frac{4}{3d-e}$ e) $\frac{5}{x+1} - \frac{2}{x+2}$ f) $\frac{3-a}{x} + \frac{2-a}{y}$

7. a) $\frac{3x+4y}{2a-b} - \frac{5x+y}{a+3b}$ b) $\frac{2a+3b-4c}{4} - \frac{a-3b+5c}{5}$
 c) $\frac{2x-3y}{10m+3n} + \frac{4x-2y}{5m-2n}$ d) $\frac{a+2}{x+1} - \frac{a+5}{x+2}$
 e) $\frac{5x+3}{2x-4} + \frac{2x-1}{x+3}$ f) $\frac{2x+5y-z}{3} + \frac{3x-4y+2z}{5}$

8. a) $\frac{x^2}{x^2-9} + \frac{x}{x+3} - \frac{x}{x-3}$ b) $\frac{5}{a-b} + \frac{3}{a^2-b^2} + 1$
 c) $\frac{a}{2(a+b)} - \frac{b}{3(a-b)} + \frac{ab}{a^2-b^2}$ d) $\frac{4x}{4x-6y} + \frac{2x}{6x+9y} - \frac{3x}{24x^2-54y^2}$



1. Bestimme die Definitionsmenge:

a) $\frac{3x - 5}{18x}$	b) $\frac{1}{4x^2 - 9}$
c) $\frac{7}{x - 3,5}$	d) $\frac{35x - 5}{8x - 2}$
e) $\frac{8x^2 + 6x - 3}{25x^2 + 20x + 4}$	f) $\frac{6x - 4}{(6x - 2)(5x + 1)}$

2. Kürze soweit wie möglich:

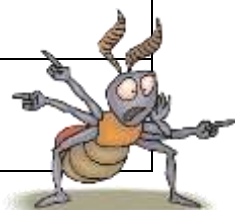
a) $\frac{15x^2}{21x}$	b) $\frac{21(x + 2)}{35(x + 2)}$
c) $\frac{8x^2 - 4x}{32x + 24x^2}$	d) $\frac{x - 5}{x^2 - 10x + 25}$
e) $\frac{14x + 6}{49x^2 + 42x + 9}$	f) $\frac{x^2 - 4y^2}{x^2 - 4xy + 4y^2}$

3. Berechne:

a) $\frac{14}{2a} - \frac{11}{2a} + \frac{3}{2a}$
b) $\frac{3x + 5}{x - 2} - \frac{2x - 6}{x - 2}$
c) $\frac{24}{4x} + \frac{42}{3x} + \frac{7}{6x}$
d) $\frac{5x + 8}{2x} + \frac{7x^2 - 2x}{11x^2}$

4. Berechne, kürze - wenn möglich - vor dem Multiplizieren:

a) $\frac{56x^2}{9} \cdot \frac{15}{48x}$
b) $\frac{108a^2}{91} \cdot \frac{39a^2}{144a}$
c) $\frac{36a}{35} : \frac{24a^2}{63a}$
d) $\frac{88x^2}{62} : \frac{132x^3}{21}$



1. Für welche Zahlen kann man den Wert des Terms nicht berechnen?

	1	4	0	-2	0,5	2,5
$\frac{3x}{x-4}$						
$\frac{x-1}{x+2}$						
$\frac{6}{2x-2}$						
$\frac{x+3}{2x-1}$						

2. Bestimme die Definitionsmenge

$$\frac{5}{2x-2} =$$

$$\frac{1}{3x-12} =$$

$$\frac{x}{5x+10} =$$

$$\frac{x+1}{8x-48} =$$

$$\frac{1-x}{100+20x} =$$

$$\frac{3}{4x-1} =$$

$$\frac{1+x}{6x+3} =$$

$$\frac{x^2}{10x-4} =$$

$$\frac{x}{45+75x} =$$

3. Gib verschiedene Bruchterme mit der vorgegebenen Definitionsmenge an

$$D = \mathbb{Q} \setminus \{1\}$$

$$D = \mathbb{Q} \setminus \{1; -1\}$$

$$D = \mathbb{Q} \setminus \{0\}$$

$$D = \mathbb{Q} \setminus \{0; 2\}$$

$$D = \mathbb{Q} \setminus \{3; -3\}$$

$$D = \mathbb{Q} \setminus \{5; -4\}$$

4. Vereinfache die Bruchterme:

$$\frac{3+6a}{3+9a} =$$

$$\frac{16+4a}{4a} =$$

$$\frac{xy+y}{zy+y} =$$

$$\frac{x+xy}{x} =$$

$$\frac{5b+25}{5b} =$$

$$\frac{2x+2y}{3x+3y} =$$

5. Kürze folgende Bruchterme so weit wie möglich!

$$\frac{14a}{7} =$$

$$\frac{32ef}{64e} =$$

$$\frac{3a \cdot 4b \cdot 5c}{6a \cdot 2c} =$$

$$\frac{40abc}{5a} =$$

$$\frac{63rs}{7s} =$$

$$\frac{3a \cdot 4b}{4a} =$$

$$\frac{12a}{-ab} =$$

$$\frac{7x \cdot 2y}{21xy} =$$

$$\frac{x}{x(3+y)} =$$



1. Berechne das Produkt

$$\frac{2x}{3y} \cdot \frac{8x}{7y} = \underline{\hspace{2cm}}$$

$$\frac{9a}{4x} \cdot \frac{8x}{18} = \underline{\hspace{2cm}}$$

$$\frac{6b}{a} \cdot 4c = \underline{\hspace{2cm}}$$

$$8b \cdot \frac{5a}{10b} = \underline{\hspace{2cm}}$$

$$\frac{4x}{5y} \cdot \frac{3z}{8} = \underline{\hspace{2cm}}$$

$$\frac{4a}{3b} \cdot \frac{(-6b)}{8} = \underline{\hspace{2cm}}$$

2. Berechne

$$\frac{14}{2a} - \frac{11}{2a} + \frac{3}{2a} = \underline{\hspace{2cm}}$$

$$\frac{3x+5}{x-2} - \frac{2x-6}{x-2} = \underline{\hspace{2cm}}$$

$$\frac{x^2+1}{x+1} - (x-1) = \underline{\hspace{2cm}}$$

$$\frac{x+3}{x-3} - \frac{x-3}{x+3} - \frac{36}{x^2-9} = \underline{\hspace{2cm}}$$

3. Berechne

$$\frac{4x}{5y} \cdot \frac{15}{2x} = \underline{\hspace{2cm}}$$

$$\frac{10x^2}{y^2} \cdot \frac{2y^2}{5x} = \underline{\hspace{2cm}}$$

$$\frac{6x}{7(x+y)} \cdot \frac{14(x+y)}{x} = \underline{\hspace{2cm}}$$

$$\frac{9(a+b)}{8a} \cdot \frac{4a^2}{18(a+b)} = \underline{\hspace{2cm}}$$

$$24mn : \frac{12n}{15m} = \underline{\hspace{2cm}}$$

$$\frac{3d}{2e} : \frac{12d}{4e} = \underline{\hspace{2cm}}$$

$$\frac{5(x^2-9y^2)}{a} : \frac{x+3y}{2a} = \underline{\hspace{2cm}}$$

$$\frac{\frac{a}{x+y}}{\frac{2a}{x^2-y^2}} = \underline{\hspace{2cm}}$$

4. Berechne den Bruchterm für folgende Belegung: x = 5; y = -3; z = -1

$$\frac{3x+5z}{x^2-2y^2}$$

5. Kürze folgende Bruchterme soweit wie möglich

$$\frac{9x-3}{3} = \underline{\hspace{2cm}}$$

$$\frac{x+1}{-x-1} = \underline{\hspace{2cm}}$$

$$\frac{3x}{x^2+4x} = \underline{\hspace{2cm}}$$

$$\frac{16x-28}{12x} = \underline{\hspace{2cm}}$$

$$\frac{x-1}{x^2-1} = \underline{\hspace{2cm}}$$

$$\frac{(2x+3)^2}{4x^2+12x+9} = \underline{\hspace{2cm}}$$

6. Ermittle den fehlenden Zähler bzw. Nenner.

$$\frac{x+2}{x-2} = \frac{x^2+4x+4}{\hspace{2cm}}$$

$$\frac{3x+2}{x+3} = \frac{\hspace{2cm}}{9-x^2}$$

7. Fasse folgende Bruchterme soweit wie möglich zusammen.

Bestimme zuvor den Definitionsbereich:

$$\frac{3x-1}{4} + \frac{x+1}{4} =$$

$$\frac{x}{2x-4} - \frac{3(x+1)}{2x-4} =$$

$$\frac{1}{x-1} + \frac{1}{x+1} =$$



1. Gib einen Bruchterm mit der Variablen x an, der für alle $x \in \mathbb{Q}$ definiert ist.

Gib einen Bruchterm mit den Variablen a und b an, der nicht definiert ist, wenn man für a die Gegenzahl von b einsetzt.

2. Kürze die folgenden Bruchterme soweit wie möglich. Gib die Definitionsmenge so an, dass ungekürzter und gekürzter Bruchterm äquivalent sind.

$$\frac{6a-3}{7-14a} =$$

$$\frac{1-4b^2}{3+6b} =$$

$$\frac{9x^2+6x+1}{18x^2-2} =$$

3. Bringe die Brüche auf den kleinsten gemeinsamen Nenner und vereinfache.

$$\frac{2a}{a-3} + \frac{2-a}{2a-6} - \frac{1}{2a} =$$

$$\frac{4x+9y}{4x^2-12xy+9y^2} - \frac{7}{2x-3y} - \frac{5}{3y-2x} =$$

4. Gib die Definitionsmenge an.

$$\frac{x+5}{2x}$$

$$\frac{a+5}{4a\left(a-\frac{1}{2}\right)}$$

$$\frac{x+22}{(4x+5)(3x-2)}$$

$$\frac{x^2-4x+4}{2x+x^2} =$$

$$\frac{2x}{4x^2-9}$$

5. Berechne und vereinfache.

$$\frac{x-y}{x^2-y^2} + \frac{xy}{y^3-x^2y} =$$

$$\frac{a^2-b^2}{2a+2b} - \frac{3a^2+4ab}{2ba-4a^2} =$$

$$\frac{x}{x-y} - 1 =$$

$$\frac{3}{x-2} + \frac{6}{x+4} =$$

$$\frac{3a^2+ab}{a^2-b^2} - \frac{2a}{a-b} =$$

$$\frac{a+2b-1}{2ab+a^2+b^2} - \frac{a}{a^2+ab} =$$



1. a) $\frac{x}{5} - \frac{y}{10}$
 $= \frac{2x}{10} - \frac{y}{10}$
 $= \frac{2x - y}{10}$

b) $\frac{5}{7d} + \frac{8}{14d}$
 $= \frac{5}{7d} + \frac{4}{7d}$
 $= \frac{9}{7d}$

c) $\frac{5}{a^2} + \frac{3}{a}$
 $= \frac{5}{a^2} + \frac{3a}{a^2}$
 $= \frac{5 + 3a}{a^2}$

d) $\frac{a}{b^3} - \frac{c}{b^2}$
 $= \frac{a}{b^3} - \frac{bc}{b^3}$
 $= \frac{a - bc}{b^3}$

e) $\frac{x}{3k} - \frac{y}{6k}$
 $= \frac{2x}{6k} - \frac{y}{6k}$
 $= \frac{2x - y}{6k}$

f) $\frac{2x}{y^2z^2} + \frac{3}{yz}$
 $= \frac{2x}{y^2z^2} + \frac{3yz}{y^2z^2}$
 $= \frac{2x + 3yz}{y^2z^2}$

g) $\frac{a}{7x} + \frac{b}{14x}$
 $= \frac{2a}{14x} + \frac{b}{14x}$
 $= \frac{2a + b}{14x}$

h) $\frac{4}{x} - \frac{2}{x^2}$
 $= \frac{4x}{x^2} - \frac{2}{x^2}$
 $= \frac{4x - 2}{x^2}$

2. a) $\frac{5x}{2} + \frac{3y}{6} - \frac{2z}{3}$
 $= \frac{15x}{6} + \frac{3y}{6} - \frac{4z}{6}$
 $= \frac{15x + 3y - 4z}{6}$

b) $\frac{5}{4x} + \frac{3}{5x} - \frac{7}{20x}$
 $= \frac{25}{20x} + \frac{12}{20x} - \frac{7}{20x}$
 $= \frac{30}{20x} = \frac{3}{2x}$

c) $\frac{5}{x^2} - \frac{2}{x} + \frac{3}{x^3}$
 $= \frac{5x}{x^3} - \frac{2x^2}{x^3} + \frac{3}{x^3}$
 $= \frac{5x - 2x^2 + 3}{x^3}$

d) $\frac{4a^2}{9} + \frac{2b^2}{6} - \frac{c^2}{18}$
 $= \frac{8a^2}{18} + \frac{6b^2}{18} - \frac{c^2}{18}$
 $= \frac{8a^2 + 6b^2 - c^2}{18}$

e) $\frac{4}{8d} - \frac{2}{6d} + \frac{9}{24d^2}$
 $= \frac{12d}{24d^2} - \frac{8d}{24d^2} + \frac{9}{24d^2}$
 $= \frac{12d - 8d + 9}{24d^2} = \frac{4d + 9}{24d^2}$

f) $\frac{3}{9m} - \frac{2}{18mn} + \frac{3}{6n}$
 $= \frac{6n}{18mn} - \frac{2}{18mn} + \frac{9m}{18mn}$
 $= \frac{6n - 2 + 9m}{18mn}$

3. a) $\frac{x+1}{3} - \frac{2x-4}{2} + \frac{3x-2}{6}$
 $= \frac{2x+2}{6} - \frac{6x-12}{6} + \frac{3x-2}{6}$
 $= \frac{-x+12}{6}$

b) $\frac{3a-4b}{5} - \frac{8a-7b}{2} + \frac{3b-4a}{10}$
 $= \frac{6a-8b}{10} - \frac{40a-35b}{10} + \frac{3b-4a}{10}$
 $= \frac{-38a+30b}{10}$

c) $\frac{5a}{a^2-b^2} - \frac{3}{a+b} + \frac{5}{a-b}$
 $= \frac{5a}{a^2-b^2} - \frac{3a-3b}{a^2-b^2} + \frac{5a+5b}{a^2-b^2}$
 $= \frac{7a+8b}{a^2-b^2}$

d) $\frac{8}{4x^2-9y^2} - \frac{2}{2x-3y} + \frac{5}{2x+3y}$
 $= \frac{8}{4x^2-9y^2} - \frac{4x+6y}{4x^2-9y^2} + \frac{10x-15y}{4x^2-9y^2}$
 $= \frac{8+6x-21y}{4x^2-9y^2}$

e) $\frac{5}{a^2-b^2} + \frac{3}{a-b} - \frac{2}{a+b}$
 $= \frac{5}{a^2-b^2} + \frac{3a+3b}{a^2-b^2} - \frac{2a-2b}{a^2-b^2}$
 $= \frac{5+a+5b}{a^2-b^2}$

f) $\frac{4}{p^2-q^2} + \frac{4}{p+q} - \frac{2}{p-q}$
 $= \frac{4}{p^2-q^2} + \frac{4p-4q}{p^2-q^2} - \frac{2p+2q}{p^2-q^2}$
 $= \frac{4+2p-6q}{p^2-q^2}$

$$\begin{aligned}
 4. \quad a) \quad & \frac{x}{a} - \frac{y}{b} \\
 &= \frac{bx}{ab} - \frac{ay}{ab} \\
 &= \frac{bx - ay}{ab}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & 5 + \frac{1}{r} \\
 &= \frac{5r}{r} + \frac{1}{r} \\
 &= \frac{5r + 1}{r}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{x}{m} + \frac{y}{n} \\
 &= \frac{xn}{mn} + \frac{ym}{mn} \\
 &= \frac{xn + ym}{mn}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{x}{y} - 1 \\
 &= \frac{x}{y} - \frac{y}{y} \\
 &= \frac{x - y}{y}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{7}{k} + 4 \\
 &= \frac{7}{k} + \frac{4k}{k} \\
 &= \frac{7 + 4k}{k}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{1}{p} - \frac{1}{q} \\
 &= \frac{q}{pq} - \frac{p}{pq} \\
 &= \frac{q - p}{pq}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad a) \quad & \frac{3}{x} + \frac{2}{y} - \frac{1}{z} \\
 &= \frac{3yz}{xyz} + \frac{2xz}{xyz} - \frac{xy}{xyz} \\
 &= \frac{3yz + 2xz - xy}{xyz}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{2x}{5p} + \frac{3y}{4q} - \frac{5z}{r} \\
 &= \frac{8qrx}{20pqr} + \frac{15pqr}{20pqr} - \frac{100pqz}{20pqr} \\
 &= \frac{8qrx + 15pqr - 100pqz}{20pqr}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{4a}{3} + \frac{5b}{2} - \frac{7}{8} \\
 &= \frac{32a}{24} + \frac{60b}{24} - \frac{21}{24} \\
 &= \frac{32a + 60b - 21}{24}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{5x}{8} - 1 + y \\
 &= \frac{5x}{8} - \frac{8}{8} + \frac{8y}{8} \\
 &= \frac{5x - 8 + 8y}{8}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{7}{x} + \frac{2}{y} - 1 \\
 &= \frac{7y}{xy} + \frac{2x}{xy} - \frac{xy}{xy} \\
 &= \frac{7y + 2x - xy}{xy}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{2a}{5} + \frac{2b}{3} - \frac{c}{6} \\
 &= \frac{12a}{30} + \frac{20b}{30} - \frac{5c}{30} \\
 &= \frac{12a + 20b - 5c}{30}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad a) \quad & \frac{2a + 3b}{5} - \frac{3a - 5b}{4} \\
 &= \frac{8a + 12b}{20} - \frac{15a - 25b}{20} \\
 &= \frac{-7a + 37b}{20}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \frac{5}{2d - e} + \frac{4}{3d - e} \\
 &= \frac{15d - 5e}{(2d - e)(3d - e)} + \frac{8d - 4e}{(2d - e)(3d - e)} \\
 &= \frac{23d - 9e}{6d^2 - 5de + e^2}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \frac{5x - 3y}{2a} - \frac{2x + 3y}{b} \\
 &= \frac{5bx - 3by}{2ab} - \frac{4ax + 6ay}{2ab} \\
 &= \frac{5bx - 3by - 4ax - 6ay}{2ab}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \frac{5}{x + 1} - \frac{2}{x + 2} \\
 &= \frac{5x + 10}{(x + 1)(x + 2)} - \frac{2x + 2}{(x + 1)(x + 2)} \\
 &= \frac{3x + 8}{x^2 + 3x + 2}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \frac{a}{x + 1} - \frac{b}{x - 1} \\
 &= \frac{ax - a}{x^2 - 1} - \frac{bx + b}{x^2 - 1} \\
 &= \frac{ax - a - bx - b}{x^2 - 1}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \frac{3 - a}{x} + \frac{2 - a}{y} \\
 &= \frac{3y - ay}{xy} + \frac{2x - ax}{xy} \\
 &= \frac{3y - ay + 2x - ax}{xy}
 \end{aligned}$$

7. a) $\frac{3x+4y}{2a-b} - \frac{5x+y}{a+3b}$

$$\begin{aligned} &= \frac{(3x+4y)(a+3b)}{(2a-b)(a+3b)} - \frac{(5x+y)(2a-b)}{(2a-b)(a+3b)} \\ &= \frac{3ax+4ay+9bx+12by - (10ax+2ay-5bx-by)}{(2a-b)(a+3b)} \\ &= \frac{3ax+4ay+9bx+12by-10ax-2ay+5bx+by}{(2a-b)(a+3b)} \\ &= \frac{-7ax+14ay+14bx+13by}{(2a-b)(a+3b)} \end{aligned}$$

b) $\frac{2a+3b-4c}{4} - \frac{a-3b+5c}{5}$

$$\begin{aligned} &= \frac{10a+15b-20c}{20} - \frac{4a-12b+20c}{20} \\ &= \frac{6a+27b-40c}{20} \end{aligned}$$

c) $\frac{2x-3y}{10m+3n} + \frac{4x-2y}{5m-2n}$

$$\begin{aligned} &= \frac{(2x-3y)(5m-2n)}{(10m+3n)(5m-2n)} + \frac{(4x-2y)(10m+3n)}{(10m+3n)(5m-2n)} \\ &= \frac{50mx+8nx-35my}{50m^2-5mn-6n^2} \end{aligned}$$

e) $\frac{5x+3}{2x-4} + \frac{2x-1}{x+3}$

$$\begin{aligned} &= \frac{(5x+3)(x+3)}{(2x-4)(x+3)} + \frac{(2x-1)(2x-4)}{(2x-4)(x+3)} \\ &= \frac{9x^2+8x+13}{2x^2+2x-12} \end{aligned}$$

d) $\frac{a+2}{x+1} - \frac{a+5}{x+2}$

$$\begin{aligned} &= \frac{(a+2)(x+2)}{(x+1)(x+2)} - \frac{(a+5)(x+1)}{(x+1)(x+2)} \\ &= \frac{a-3x-1}{x^2+3x+2} \end{aligned}$$

f) $\frac{2x+5y-z}{3} + \frac{3x-4y+2z}{5}$

$$\begin{aligned} &= \frac{10x+25y-5z}{15} + \frac{9x-12y+6z}{15} \\ &= \frac{19x+13y+z}{15} \end{aligned}$$

8. a) $\frac{x^2}{x^2-9} + \frac{x}{x+3} - \frac{x}{x-3}$

$$\begin{aligned} &= \frac{x^2}{x^2-9} + \frac{x(x-3)}{x^2-9} - \frac{x(x+3)}{x^2-9} \\ &= \frac{x^2-6x}{x^2-9} \end{aligned}$$

c) $\frac{a}{2(a+b)} - \frac{b}{3(a-b)} + \frac{ab}{a^2-b^2}$

$$\begin{aligned} &= \frac{3a(a-b)}{6a^2-6b^2} - \frac{2b(a+b)}{6a^2-6b^2} + \frac{6ab}{6a^2-6b^2} \\ &= \frac{3a^2+ab-2b^2}{6a^2-6b^2} \end{aligned}$$

b) $\frac{5}{a-b} + \frac{3}{a^2-b^2} + 1$

$$\begin{aligned} &= \frac{5a+5b}{a^2-b^2} + \frac{3}{a^2-b^2} + \frac{a^2-b^2}{a^2-b^2} \\ &= \frac{5a+5b+3+a^2-b^2}{a^2-b^2} \end{aligned}$$

d) $\frac{4x}{4x-6y} + \frac{2x}{6x+9y} - \frac{3x}{24x^2-54y^2}$

$$\begin{aligned} &= \frac{4x}{2(2x-3y)} + \frac{2x}{3(2x+3y)} - \frac{3x}{6(4x^2-9y^2)} \\ &= \frac{12x(2x+3y)}{24x^2-54y^2} + \frac{4x(2x-3y)}{24x^2-54y^2} - \frac{3x}{24x^2-54y^2} \\ &= \frac{32x^2+24xy-3x}{24x^2-54y^2} \end{aligned}$$

1. Bestimme die Definitionsmenge: (!Nenner darf nicht 0 sein!)

a) $\frac{3x-5}{18x}$ $D = \mathbb{Q} \setminus \{0\}$	b) $\frac{1}{4x^2-9}$ $D = \mathbb{Q} \setminus \{\frac{3}{2}; -\frac{3}{2}\}$ $\frac{1}{(2x-3)(2x+3)}$
c) $\frac{7}{x-3,5}$ $D = \mathbb{Q} \setminus \{3,5\}$	d) $\frac{35x-5}{8x-2}$ $D = \mathbb{Q} \setminus \{\frac{1}{4}\}$
e) $\frac{8x^2+6x-3}{25x^2+20x+4}$ $D = \mathbb{Q} \setminus \{-2\}$ $\frac{8x^2+6x-3}{(5x+2)^2}$	f) $\frac{6x-4}{(6x-2)(5x+1)}$ $D = \mathbb{Q} \setminus \{\frac{1}{3}; -\frac{1}{5}\}$

2. Kürze soweit wie möglich:

a) $\frac{15x^2}{21x} = \frac{5x}{7}$	b) $\frac{21(x+2)}{35(x+2)} = \frac{21}{35} = \frac{3}{5}$
c) $\frac{8x^2-4x}{32x+24x^2} = \frac{4x(2x-1)}{8x(4+3x)} = \frac{2x-1}{2(4+3x)}$	d) $\frac{x-5}{x^2-10x+25} = \frac{x-5}{(x-5)^2} = \frac{1}{(x-5)}$
e) $\frac{14x+6}{49x^2+42x+9} = \frac{2(7x+3)}{(7x+3)^2} = \frac{2}{7x+3}$	f) $\frac{x^2-4y^2}{x^2-4xy+4y^2} = \frac{(x-2y)(x+2y)}{(x-2y)^2} = \frac{(x+2y)}{(x-2y)}$

3. Berechne

a) $\frac{14}{2a} - \frac{11}{2a} + \frac{3}{2a} = \frac{14-11+3}{2a} = \frac{6}{2a} = \frac{3}{a}$
b) $\frac{3x+5}{x-2} - \frac{2x-6}{x-2} = \frac{3x+5-(2x-6)}{x-2} = \frac{3x+5-2x+6}{x-2} = \frac{x+11}{x-2}$
c) $\frac{24}{4x} + \frac{42}{3x} + \frac{7}{6x} = \frac{24 \cdot 3}{4x \cdot 3} + \frac{42 \cdot 4}{3x \cdot 4} + \frac{7 \cdot 2}{6x \cdot 2} = \frac{72}{12x} + \frac{168}{12x} + \frac{14}{12x} = \frac{254}{12x} = \frac{127}{6x}$
d) $\frac{5x+8}{2x} + \frac{x(7x-2)}{11x^2} = \frac{5x+8}{2x} + \frac{7x-2}{11x} = \frac{(5x+8) \cdot 11}{2x \cdot 11} + \frac{(7x-2) \cdot 2}{11x \cdot 2} = \frac{55x+88}{22x} + \frac{14x-4}{22x} = \frac{55x+88+14x-4}{22x} = \frac{69x+84}{22x}$

4. Berechne, kürze - wenn möglich - vor dem Multiplizieren:

a) $\frac{56x^2}{9} \cdot \frac{15}{48x} = \frac{7x}{3} \cdot \frac{5}{6} = \frac{35x}{18} = \frac{35}{18}x$
b) $\frac{108a^2}{91} \cdot \frac{39a^2}{144a} = \frac{3a}{7} \cdot \frac{3a^2}{4} = \frac{9a^3}{28} = \frac{9}{28}a^3$
c) $\frac{36a}{35} : \frac{24a^2}{63a} = \frac{36a}{35} \cdot \frac{63a}{24a^2} = \frac{3}{5} \cdot \frac{9}{2} = \frac{27}{10}$
d) $\frac{88x^2}{62} : \frac{132x^3}{21} = \frac{88x^2}{62} \cdot \frac{21}{132x^3} = \frac{2}{3} \cdot \frac{1}{3x} = \frac{2}{9x}$

1. Für welche Zahlen kann man den Wert des Terms nicht berechnen?

	1	4	0	-2	0,5	2,5
$\frac{3x}{x-4}$	-1	/	0	1	$-\frac{3}{7}$	-5
$\frac{x-1}{x+2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	/	-0,2	$\frac{1}{3}$
$\frac{6}{2x-2}$	/	1	-3	-1	-6	2
$\frac{x+3}{2x-1}$	4	1	-3	$-\frac{1}{5}$	/	1,375

2. Bestimme die Definitionsmenge

$$\frac{5}{2x-2} : D = \mathbb{Q} \setminus \{1\}$$

$$\frac{1}{3x-12} : D = \mathbb{Q} \setminus \{4\}$$

$$\frac{x}{5x+10} : D = \mathbb{Q} \setminus \{-2\}$$

$$\frac{x+1}{8x-48} : D = \mathbb{Q} \setminus \{6\}$$

$$\frac{1-x}{100+20x} : D = \mathbb{Q} \setminus \{-5\}$$

$$\frac{3}{4x-1} : D = \mathbb{Q} \setminus \left\{\frac{1}{4}\right\}$$

$$\frac{1+x}{6x+3} : D = \mathbb{Q} \setminus \left\{-\frac{1}{2}\right\}$$

$$\frac{x^2}{10x-4} : D = \mathbb{Q} \setminus \left\{\frac{2}{5}\right\}$$

$$\frac{x}{45+75x} : D = \mathbb{Q} \setminus \left\{-\frac{3}{5}\right\}$$

3. Gib verschiedene Bruchterme mit der vorgegebenen Definitionsmenge an

$$D = \mathbb{Q} \setminus \{1\}$$

$$\frac{1}{1-x}$$

$$D = \mathbb{Q} \setminus \{1; -1\}$$

$$\frac{1}{x^2-1}$$

$$D = \mathbb{Q} \setminus \{0\}$$

$$\frac{1}{x}; \frac{1}{x^2}$$

$$D = \mathbb{Q} \setminus \{0; 2\}$$

$$\frac{1}{x(x-2)}$$

$$D = \mathbb{Q} \setminus \{3; -3\}$$

$$\frac{1}{(x+3)(x-3)}$$

$$D = \mathbb{Q} \setminus \{5; -4\}$$

$$\frac{1}{(x-5)(x+4)}$$

4. Vereinfache die Bruchterme:

$$\frac{3+6a}{3+9a} = \frac{3(1+2a)}{3(1+3a)} = \frac{1+2a}{1+3a}$$

$$\frac{16+4a}{4a} = \frac{4(4+a)}{4a} = \frac{4+a}{a}$$

$$\frac{xy+y}{zy+y} = \frac{y(x+1)}{y(z+1)} = \frac{x+1}{z+1}$$

$$\frac{x+xy}{x} = \frac{x(1+y)}{x} = 1+y$$

$$\frac{5b+25}{5b} = \frac{5(b+5)}{5b} = \frac{b+5}{b}$$

$$\frac{2x+2y}{3x+3y} = \frac{2(x+y)}{3(x+y)} = \frac{2}{3}$$

5. Kürze folgende Bruchterme so weit wie möglich!

$$\frac{14a}{7} = 2a$$

$$\frac{32ef}{64e} = \frac{f}{2} = \frac{1}{2}f$$

$$\frac{3a \cdot 4b \cdot 5c}{6a \cdot 2c} = \frac{60abc}{12ac} = 5b$$

$$\frac{40abc}{5a} = 8bc$$

$$\frac{63rs}{7s} = 9r$$

$$\frac{3a \cdot 4b}{4a} = 3b$$

$$\frac{12a}{-ab} = -\frac{4}{b}$$

$$\frac{7x \cdot 2y}{21xy} = \frac{2}{3}$$

$$\frac{x}{x \cdot (3+y)} = \frac{1}{3+y}$$

1. Berechne das Produkt

$$\frac{2x}{3y} \cdot \frac{8x}{7y} = \frac{2x \cdot 8x}{3y \cdot 7y} = \frac{16x^2}{21y^2}$$

$$\frac{9a}{4x} \cdot \frac{8x}{18} = \frac{9a \cdot 8x}{4x \cdot 18} = \frac{2a}{2} = a$$

$$\frac{6b}{a} \cdot 4c = \frac{6b \cdot 4c}{a} = \frac{24bc}{a}$$

$$8b \cdot \frac{5a}{10b} = \frac{8b \cdot a}{2b} = 4a$$

$$\frac{4x}{5y} \cdot \frac{3z}{8} = \frac{4x \cdot 3z}{5y \cdot 8} = \frac{3xz}{10y}$$

$$\frac{4a}{3b} \cdot \frac{(-6b)}{8} = \frac{4a \cdot (-6b)}{3b \cdot 8} = \frac{-ab}{b} = -a$$

2. Berechne

$$\frac{14}{2a} - \frac{11}{2a} + \frac{3}{2a} = \frac{14-11+3}{2a} = \frac{6}{2a} = \frac{3}{a}$$

$$\frac{3x+5}{x-2} - \frac{2x-6}{x-2} = \frac{3x+5-2x+6}{x-2} = \frac{x+11}{x-2}$$

$$\frac{x^2+1}{x+1} - (x-1) = \frac{x^2+1-(x-1)(x+1)}{x+1} = \frac{x^2+1-x^2+1}{x+1} = \frac{2}{x+1}$$

$$\frac{x+3}{x-3} - \frac{x-3}{x+3} - \frac{36}{x^2-9} = \frac{(x+3)^2 - (x-3)^2 - 36}{(x+3)(x-3)} = \frac{x^2+6x+9-x^2+6x-9-36}{(x+3)(x-3)} = \frac{12x-36}{(x+3)(x-3)} = \frac{12(x-3)}{(x+3)(x-3)} = \frac{12}{x+3}$$

3. Berechne

$$\frac{4x}{5y} \cdot \frac{15}{2x} = \frac{2 \cdot 3}{y} = \frac{6}{y}$$

$$\frac{10x^2}{y^2} \cdot \frac{2y^2}{5x} = \frac{2x \cdot 2}{1} = 4x$$

$$\frac{6x}{7(x+y)} \cdot \frac{14(x+y)}{x} = \frac{6 \cdot 2}{1} = 12$$

$$\frac{9(a+b)}{8a} \cdot \frac{4a^2}{18(a+b)} = \frac{a}{2 \cdot 2} = \frac{a}{4}$$

$$24mn : \frac{12n}{15m} = \frac{24mn \cdot 15m}{12n} = 30m^2$$

$$\frac{3d}{2e} : \frac{12d}{4e} = \frac{3d \cdot 4e}{2e \cdot 12d} = \frac{1}{2}$$

$$\frac{5(x^2-9y^2)}{a} : \frac{x+3y}{2a} = \frac{5(x^2-9y^2) \cdot 2a}{a(x+3y)} = 10(x-3y)$$

$$\frac{\frac{x+y}{2a}}{x^2-y^2} = \frac{a \cdot (x^2-y^2)}{(x+y) \cdot 2a} = \frac{x-y}{2}$$

4. Berechne den Bruchterm für folgende Belegung: x = 5; y = -3; z = -1

$$\frac{3x+5z}{x^2-2y^2} = \frac{3 \cdot 5 + 5 \cdot (-1)}{5 \cdot 5 - 2 \cdot (-3) \cdot (-3)} = \frac{15-5}{25-18} = \frac{10}{7} = 1 \frac{3}{7}$$

5. Kürze folgende Bruchterme soweit wie möglich

$$\frac{9x-3}{3} = \frac{3(3x-1)}{3} = 3x-1$$

$$\frac{x+1}{-x-1} = \frac{x+1}{(-1)(x+1)} = -1$$

$$\frac{3x}{x^2+4x} = \frac{3x}{x(x+4)} = \frac{3}{x+4}$$

$$\frac{16x-28}{12x} = \frac{4(4x-7)}{12x} = \frac{4x-7}{3x}$$

$$\frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{x}{x+1}$$

$$\frac{(2x+3)^2}{4x^2+12x+9} = \frac{(2x+3)^2}{(2x+3)^2} = 1$$

6. Ermittle den fehlenden Zähler bzw. Nenner.

$$\frac{x+2}{x-2} = \frac{(x+2)(x+2)}{(x-2)(x+2)} = \frac{x^2 + 4x + 4}{x^2 - 4}$$

$$\frac{3x+2}{x+3} = \frac{(3x+2)(x-3)}{(x+3)(x-3)} = \frac{9x+6-3x^2-2x}{9-x^2} = \frac{-3x^2+7x+6}{9-x^2}$$

7. Fasse folgende Bruchterme soweit wie möglich zusammen.

Bestimme zuvor den Definitionsbereich:

$$\frac{3x-1}{4} + \frac{x+1}{4} = \frac{3x-1+x+1}{4} = \frac{4x}{4} = x$$

$$D = x \in \mathbb{Q}$$

$$\frac{x}{2x-4} - \frac{3(x+1)}{2x-4} = \frac{-2x-3}{2x-4}$$

$$D = x \in \mathbb{Q} \setminus \{2\}$$

$$\frac{1}{x-1} + \frac{1}{x+1} = \frac{x+1}{(x-1)(x+1)} + \frac{x-1}{(x-1)(x+1)} = \frac{2x}{x^2-1}$$

$$D = x \in \mathbb{R} \setminus \{-1; 1\}$$

Bruchterme - Lösungen 5

1. Gib einen Bruchterm mit der Variablen x an, der für alle $x \in \mathbb{Q}$ definiert ist.

Es gibt keinen Bruchterm, der für alle $x \in \mathbb{Q}$ definiert ist, da ein Bruchterm als ein Term definiert ist, bei dem die Variable mindestens einmal im Nenner auftaucht.

Gib einen Bruchterm mit den Variablen a und b an, der nicht definiert ist, wenn man für a die Gegenzahl von b einsetzt.

$$\frac{1}{-a-b}; \text{ denn } \frac{1}{-(-b)-b} = \frac{1}{b-b} = \frac{1}{0}$$

2. Kürze die folgenden Bruchterme soweit wie möglich. Gib die Definitionsmenge so an, dass ungekürzter und gekürzter Bruchterm äquivalent sind.

$$\frac{6a-3}{7-14a} = \frac{-3(1-2a)}{7(1-2a)} = -\frac{3}{7}$$

$$D = a \in \mathbb{Q} \setminus \left\{ \frac{1}{2} \right\}$$

$$\frac{1-4b^2}{3+6b} = \frac{(1-2b)(1+2b)}{3(1+2b)} = \frac{1-2b}{3}$$

$$D = x \in \mathbb{Q} \setminus \left\{ -\frac{1}{2} \right\}$$

$$\frac{9x^2+6x+1}{18x^2-2} = \frac{(3x+1)(3x+1)}{(3x-1)(3x+1)} = \frac{3x+1}{3x-1}$$

$$D = x \in \mathbb{Q} \setminus \left\{ \frac{1}{3} \right\}$$

3. Bringe die Brüche aus den kleinsten gemeinsamen Nenner und vereinfache.

$$\frac{2a}{a-3} + \frac{2-a}{2a-6} - \frac{1}{2a} = \frac{2a}{a-3} + \frac{2-a}{2(a-3)} - \frac{1}{2a} = \frac{4a^2}{2a(a-3)} + \frac{a(2-a)}{2a(a-3)} - \frac{a-3}{2a(a-3)} =$$

$$\frac{4a^2+2a-a^2-a+3}{2a(a-3)} = \frac{3a^2+a+3}{2a(a-3)} = \frac{3a^2+a+3}{2a^2-6a}$$

$$\frac{4x+9y}{4x^2-12xy+9y^2} - \frac{7}{2x-3y} - \frac{5}{3y-2x} = \frac{4x+9y}{(2x-3y)^2} - \frac{7}{2x-3y} - \frac{5}{(-1)(2x-3y)} =$$

$$\frac{4x+9y}{(2x-3y)^2} - \frac{7(2x-3y)}{(2x-3y)^2} + \frac{5(2x-3y)}{(2x-3y)^2} = \frac{4x+9y-14x+21y+10x-15y}{(2x-3y)^2} = \frac{15y}{(2x-3y)^2}$$

4. Gib die Definitionsmenge an.

$$\frac{x+5}{2x} \quad D = x \in \mathbb{Q} \setminus \{0\}$$

$$\frac{a+5}{4a\left(a-\frac{1}{2}\right)} \quad D = a \in \mathbb{Q} \setminus \{0; \frac{1}{2}\}$$

$$\frac{x+22}{(4x+5)(3x-2)} \quad D = x \in \mathbb{Q} \setminus \{-1\frac{1}{4}, \frac{2}{3}\} \quad \frac{x^2-4x+4}{2x+x^2} = \frac{x^2-4x+4}{(x+2)x} \quad D = x \in \mathbb{Q} \setminus \{-2; 0\}$$

$$\frac{2x}{4x^2-9} = \frac{2x}{(2x+3)(2x-3)} \quad D = x \in \mathbb{Q} \setminus \{-\frac{3}{2}; \frac{3}{2}\}$$

5. Berechne und vereinfache

$$\frac{x-y}{x^2-y^2} + \frac{xy}{y^3-x^2y} = \frac{x-y}{x^2-y^2} + \frac{xy}{y(y^2-x^2)} = \frac{x-y+x}{x^2-y^2} = \frac{2x-y}{x^2-y^2}$$

$$\frac{a^2-b^2}{2a+2b} - \frac{3a^2+4ab}{2ba-4a^2} = \frac{(a-b)(a+b)}{2(a+b)} - \frac{a(3a+4b)}{2a(b-2a)} = \frac{a-b}{2} - \frac{3a+4b}{2(b-2a)} =$$

$$\frac{(a-b)(b-2a)-3a-4b}{2(b-2a)} = \frac{ab-b^2-2a^2+2ab-3a-4b}{2(b-2a)} = \frac{3ab-3a-4b-2a^2-b^2}{2(b-2a)}$$

$$\frac{x}{x-y} - 1 = \frac{x}{x-y} - \frac{x-y}{x-y} = \frac{x-x+y}{x-y} = \frac{y}{x-y}$$

$$\frac{3}{x-2} + \frac{6}{x+4} = \frac{3(x+4)+6(x-2)}{(x-2)(x+4)} = \frac{3x+12+6x-12}{(x-2)(x+4)} = \frac{9x}{(x-2)(x+4)}$$

$$\frac{3a^2+ab}{a^2-b^2} - \frac{2a}{a-b} = \frac{3a^2+ab}{(a-b)(a+b)} - \frac{2a}{a-b} = \frac{3a^2+ab-2a(a+b)}{(a-b)(a+b)} = \frac{3a^2+ab-2ab-2a^2}{(a-b)(a+b)} =$$

$$\frac{a^2-ab}{(a-b)(a+b)} = \frac{a(a-b)}{(a-b)(a+b)} = \frac{a}{a+b}$$

$$\frac{a+2b-1}{2ab+a^2+b^2} - \frac{a}{a^2+ab} = \frac{a+2b-1}{(a+b)^2} - \frac{a}{a(a+b)} = \frac{a+2b-1}{(a+b)^2} - \frac{1}{(a+b)} = \frac{a+2b-1-(a+b)}{(a+b)^2} =$$

$$\frac{a+2b-1-a-b}{(a+b)^2} = \frac{b-1}{(a+b)^2}$$