3. Mathematik-Klassenarbeit Klasse 11a

Aufgabe 1

Die Punkte $A(0|3)$, $B\left(\frac{13}{2}\middle|\frac{5}{2}\right)$ und $C\left(\frac{7}{2}|6\right)$ bilden ein Dreieck.

- Zeige, dass das Dreieck ABC rechtwinklig ist.
- Berechne den Flächeninhalt des Dreiecks ABC.

Aufgabe 2

Berechne die Grenzwerte!

a) $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}+2\right)$

b) $\lim\limits_{x\to 3}\frac{2x^2-6x}{x^2+x-12}$

Aufgabe 3

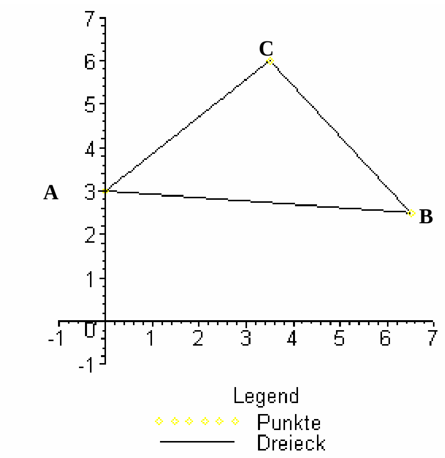
Untersuche das Verhalten der Funktion $f: x \longmapsto f(x)$ an der Definitionslücke und für $x \to \pm\infty$. Skizziere jeweils das Schaubild von f.

a) $f(x)=\frac{2x^2+10x+12}{x+3}$

b) $f(x)=\frac{x+2}{\frac{1}{4}x+1}$

Lösungsvorschlag

Aufgabe 1



$$\left.\begin{array}{l} m_{AC} = \dfrac{y_C - y_A}{x_C - x_A} = \dfrac{6-3}{3{,}5-0} = \dfrac{6}{7} \\[4em] m_{BC} = \dfrac{y_C - y_B}{x_C - x_B} = \dfrac{6-2{,}5}{3{,}5-6{,}5} = -\dfrac{7}{6} \end{array}\right\} m_{AC} = -\frac{1}{m_{BC}} \rightarrow AC \perp BC \rightarrow \text{Dreieck rechtwinklig}$$

Flächeninhalt berechnen:

$$A = \frac{1}{2}\,|\,0\cdot(2{,}5-6)+6{,}5\cdot(6-3)+3{,}5\cdot(3-2{,}5)\,| = \underline{\underline{10{,}625 FE}}$$

Aufgabe 2

a) $$\lim_{x\to\infty}\left(\frac{x}{x+1}+2\right) = \lim_{x\to\infty}\left(\frac{x+2\cdot(x+1)}{x+1}\right) = \lim_{x\to\infty}\left(\frac{3x+2}{x+1}\right) = \lim_{x\to\infty}\left(\frac{x\cdot(3+\frac{2}{x})}{x\cdot(1+\frac{1}{x})}\right) = \underline{\underline{3}}$$

b) $$\lim_{x\to 3}\left(\frac{2x^2+6x}{x^2+x-12}\right) = \lim_{n\to\infty}\left(\frac{2\cdot(3+\frac{1}{n})^2-6\cdot(3+\frac{1}{n})}{(3+\frac{1}{n})^2+(3+\frac{1}{n})-12}\right) = \lim_{n\to\infty}\left(\frac{\frac{6}{n}+\frac{2}{n^2}}{\frac{7}{n}+\frac{1}{n^2}}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}\cdot(6+\frac{2}{n})}{\frac{1}{n}\cdot(7+\frac{1}{n})}\right) = \underline{\underline{\frac{6}{7}}}$$

Aufgabe 3

a) Definitionslücke: $0 = x+3 \rightarrow \underline{x_D = -3}$

von rechts:

$$\lim_{x\to -3}\left(\frac{2x^2+10x+12}{x+3}\right) = \lim_{n\to\infty}\left(\frac{2\cdot(-3+\frac{1}{n})^2+10\cdot(-3+\frac{1}{n})+12}{(-3+\frac{1}{n})+3}\right) = \lim_{n\to\infty}\left(\frac{-\frac{2}{n}+\frac{2}{n^2}}{\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}\cdot(-2+\frac{2}{n})}{\frac{1}{n}}\right) = \underline{\underline{-2}}$$

von links:

$$\lim_{x\to -3}\left(\frac{2x^2+10x+12}{x+3}\right) = \lim_{n\to\infty}\left(\frac{2\cdot(-3-\frac{1}{n})^2+10\cdot(-3-\frac{1}{n})+12}{(-3-\frac{1}{n})+3}\right) = \lim_{n\to\infty}\left(\frac{\frac{2}{n}+\frac{2}{n^2}}{-\frac{1}{n}}\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{n}\cdot(2+\frac{2}{n})}{\frac{1}{n}\cdot(-1)}\right) = \underline{\underline{-2}}$$

Verhalten im Unendlichen:

$$\lim_{x\to +\infty}\left(\frac{2x^2+10x+12}{x+3}\right) = \lim_{x\to +\infty}\left(\frac{x^2\cdot(2+\frac{10}{x}+\frac{12}{x^2})}{x^2\cdot(\frac{1}{x}+\frac{3}{x^2})}\right) \underline{\underline{\rightarrow +\infty}}$$

$$\lim_{x\to -\infty}\left(\frac{2x^2+10x+12}{x+3}\right) = \lim_{x\to -\infty}\left(\frac{x^2\cdot(2+\frac{10}{x}+\frac{12}{x^2})}{x^2\cdot(\frac{1}{x}+\frac{3}{x^2})}\right) \underline{\underline{\rightarrow -\infty}}$$

b) Definitionslücke: $0 = \frac{1}{4}x+1 \rightarrow \underline{x_D = -4}$

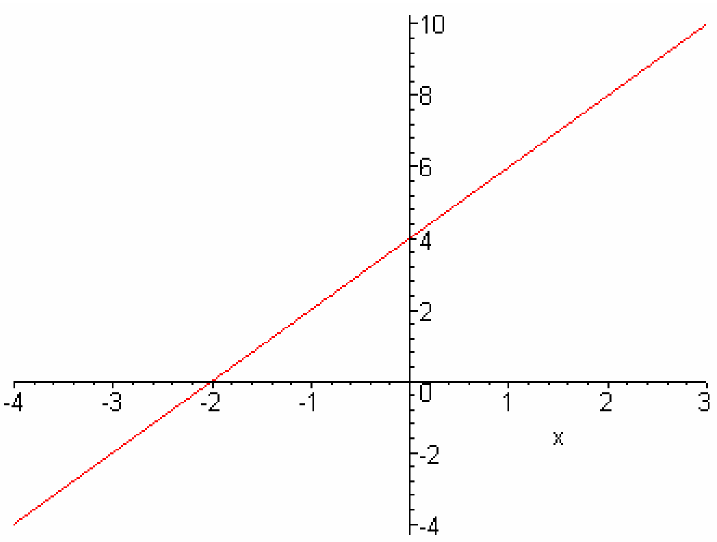
von rechts: $$\lim_{x\to -4}\left(\frac{x+2}{\frac{1}{4}x+1}\right) = \lim_{n\to\infty}\left(\frac{-4+\frac{1}{n}+2}{\frac{1}{4}(-4+\frac{1}{n})+1}\right) = \lim_{n\to\infty}\left(\frac{-2+\frac{1}{n}}{\frac{1}{4n}}\right) = \lim_{n\to\infty}\left(\frac{4n\cdot(-2+\frac{1}{n})}{1}\right) = \underline{\underline{-\infty}}$$

von links: $$\lim_{x\to -4}\left(\frac{x+2}{\frac{1}{4}x+1}\right) = \lim_{n\to\infty}\left(\frac{-4-\frac{1}{n}+2}{\frac{1}{4}(-4-\frac{1}{n})+1}\right) = \lim_{n\to\infty}\left(\frac{-2-\frac{1}{n}}{-\frac{1}{4n}}\right) = \lim_{n\to\infty}\left(\frac{-4n\cdot(-2-\frac{1}{n})}{1}\right) = \underline{\underline{+\infty}}$$

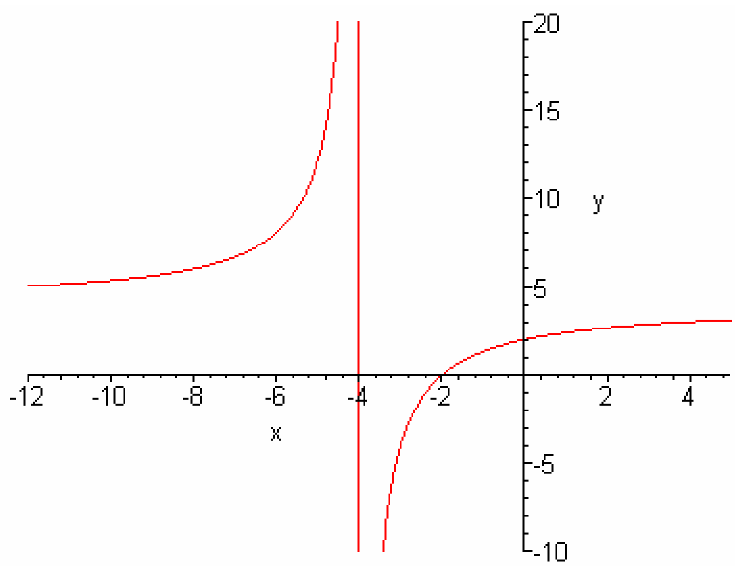
Verhalten im Unendlichen:

$$\lim_{x\to +\infty}\left(\frac{x+2}{\frac{1}{4}x+1}\right) = \lim_{x\to +\infty}\left(\frac{x\cdot(1+\frac{2}{x})}{x\cdot(\frac{1}{4}+\frac{1}{x})}\right) = \lim_{x\to +\infty}\left(\frac{1}{\frac{1}{4}}\right) = \underline{\underline{4}}$$

$$\lim_{x\to -\infty}\left(\frac{x+2}{\frac{1}{4}x+1}\right) = \lim_{x\to -\infty}\left(\frac{x\cdot(1+\frac{2}{x})}{x\cdot(\frac{1}{4}+\frac{1}{x})}\right) = \lim_{x\to -\infty}\left(\frac{1}{\frac{1}{4}}\right) = \underline{\underline{4}}$$



$$f(x)=\frac{2x^2+10x+12}{x+3}$$



$$f(x)=\frac{x+2}{\frac{1}{4}x+1}$$